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The Absence of Gravitational Deceleration during the Expansion of the Universe with increasing Mass

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Abstract
It is demonstrated that the total mass of the universe is linearly increasing with time. As a consequence, a mass moving radially with the expanding universe is not decelerated, because it finds itself, during its movement, at the same gravitational potential. The mass variation eliminates the differences between a relativistic and a Newtonian approach to describe the universe expansion, leading to final coincident equations. The principal cosmological parameters (age, size and mass of the Universe, Hubble constant) have been calculated from the microwave background radiation with an accuracy of ± 0.12 per cent. The cosmological constant and the curvature of the universe result to be null. Also the parameters of the primordial universe have been theoretically calculated with the same accuracy of the Planck length. It is suggested that it may be possible from the developed model to calculate the time of appearance of elementary particles. The mass of the particles constituting the dark matter and the time of their appearance have been estimated from the ratio dark/total matter.
Keywords: Gravitation, Cosmological parameters, Early universe

1. Introduction

The Birkhoff's theorem of the general relativity states that

$$GMR^{-1} = V_R^2/2 + A \tag{1}$$

where R is the radius of a spherical space containing the mass M (including all equivalent contributions: matter, radiation, etc.), V_R is the velocity at the distance R and A is a constant which in the FLRW metric developed by Friedmann (1922, 1924), Lemaître (1927), Robertson [1935,1936$_a$,1936$_b$) and Walker (1937) is $A=kc^2/2$, k indicating the spatial curvature, (k=+1,0,-1 for hyperspherical, flat or hyperbolic universe respectively), and c is the speed of light.

Let us consider the case in which M and R are the total mass and the radius of the universe respectively and V_R is the expansion speed at the distance R. If we apply the Birkhoff's theorem to a sphere with the same centre and with a radius s, it will be:

$$4\pi GDs^2/3 = v_s^2/2 + A = GMs^2R^{-3} \tag{2}$$

where $D=3MR^{-3}/4\pi$ is the density and v_s is the expansion speed at the distance s from the centre. Since, according to the Hubble law, it must be $v_s=V_RsR^{-1}$, combining Eq. (1) with Eq. (2) we shall obtain $A=As^2R^{-2}$ i.e. A=0. Only a flat universe (k=0) can be consistent with the Birkhoff's theorem and the Hubble law.
According to Eq. (1), since R is varying with time due to the expansion of the universe, three different cases can be considered:
(a) V_R is decreasing with time, while G and M are constant;
(b) G is increasing with time, M and V_R are constant,
(c) M is increasing with time, G and V_R are constant.
Case (a) has to be excluded. Actually, it can be easily verified that from the early stage of the universe down to an age of about 10^8 years the value of V_R calculated by Eq.(1) would be enormously higher than the speed of light and only in a recent time, when R reaches a sufficiently high value, V_R would decrease to subluminal values. Although a superluminal velocity is considered acceptable by Davis & Lineweaver (2003), it is generally excluded, according to the special relativity, that the limit value of the speed of light can be exceeded. The introduction of special relativistic corrections would prevent the recession velocity to be higher than the light speed, but it would lead to a non linear relation between H and distance even at low values of red shift, at which the Hubble law is experimentally shown to be valid. Moreover, if the velocity increases linearly up to a certain distance but remains constant beyond that point, the density isotropy of the Universe would be destroyed, because the condition for the isotropy to be maintained is the linear proportionality between velocity and

distance as established by Hubble.
Therefore we must take into consideration cases (b) and (c).
Since the value of G is strictly related to the Planck length and mass, l_p and m_p ($G=l_pc^2m_p^{-1}$), the case (b) would imply that also these parameters are not fundamental constants but would vary with time. Moreover, the values of the cosmological parameters which would be eventually obtained in this case are not acceptable.
So we shall conclude that, according to case (c), the mass M is varying with time; not only the size of the universe is expanding, but also its mass is proportionally increasing. Although the mass is increasing with time, there is no relationship between the present approach and the steady-state or quasi-steady-state model (Hoyle, 1993): in particular the density in the present study is not constant but decreasing and the big bang is maintained; therefore, all the implications of a big bang model, including the cosmic microwave background radiation (CMBR), are compatible with the present model, but the mass is demonstrated to necessarily increase linearly with time in order to respect the relativity requirement of the light speed limit.
According to the FLRW metric, the expansion rate of the universe is governed by the equations:

$$d\rho/dt=-3H(\rho+pc^{-2}) \qquad (3)$$

and

$$d^2a/dt^2 = -4\pi G(\rho+3pc^{-2})a/3 + \Lambda ac^2/3 \qquad (4)$$

where ρ is the density, t is time, H is the Hubble constant, p is the pressure, a is the scale factor and Λ is the cosmological constant. If we assume that the expansion speed varies with time and G and the total mass M (including the equivalent mass of energy) remain constant, the solution of (3) is Friedmann's equation

$$H^2 = 8\pi G\rho/3 - kc^2a^{-2} + \Lambda c^2/3 \qquad (5)$$

Eq.s (3) and (4) maintain their general validity, but it will be demonstrated in section **2** that the variation of the total mass of the universe with time leads to the vanishing of the gravitation term. The resulting model, besides explaining the absence of deceleration in the expansion process and respecting the limit value of the speed, $V_R=c$, as required by the relativity theory, the Hubble law and the isotropy of the universe, allows us to obtain values of the cosmological parameters which are in excellent agreement with the experimental data and have very low limits of uncertainty. Moreover, it will be demonstrated that, as far as the universe expansion is concerned, due to the linear variation of mass with time, both a relativistic and a Newtonian approach lead to the same conclusions.

2. The consequence of mass variation on the universe expansion

The energy conservation principle $c^2d(\rho R^3) + pd(R^3) = 0$ requires that $d\rho/dt=-3H(\rho+pc^{-2})$, (Eq. 3), and

$$pc^{-2} = -(dM/dR)R^{-2}/4\pi \qquad (6)$$

From (1), according to case (c), $dM/dR=(V_R^2G^{-1}/2+AG^{-1})= MR^{-1}$ and then

$$pc^{-2} = -MR^{-3}/4\pi=-\rho/3 \qquad (7)$$

In this case Eq. (3) would become

$$d\rho/dt=-2H\rho \qquad (8)$$

and the term $-4\pi G(\rho+3pc^{-2})a/3$ in Eq. (4), accounting for gravitational acceleration, would become zero.
The same results can be obtained from a purely Newtonian approach. Actually, being the ratio M/R constant, the density decreases with R^2, obtaining $d\rho/dt=-2H\rho$, (Eq.8). With M variable it is no more justified to use the gravitational acceleration instead of the derivative of the gravitational potential, because it would imply the constancy of the mass. Then, considering also Eq.(2),

$$dv/dt = d(GMR^{-3}s^2)/ds = d[V_R^2R^{-2}s^2/2]/ds \qquad (9)$$

The Hubble law requires sR^{-1} to be constant for any element moving with the expanding universe, because s and R increase in the same proportion during expansion, and since V_R in case (c) is constant, the right term in (9) is

zero. Actually a mass moving with the expanding universe is not subjected to the gravitation force because its gravitation potential is $v^2/2$ in every point and is constant. Consequently, there is no deceleration and no need of a cosmological constant. This result is consistent with the constancy of V_R and $v = V_R sR^{-1}$. So, assuming the linear variation of M with R, the results of the Newtonian and the relativistic approach for the expansion process coincide. The Newtonian demonstration of absence of deceleration was based on the Hubble law of expansion and a constant speed limit, while the relativistic demonstration was based on the energy conservation principle.
Then it will be

$$dv/dt = d(Hs)/dt = H^2 s + s dH/dt = 0 \tag{10}$$

Since there is no relenting effect due to gravitation, it is reasonable to assume that v at the largest distance will reach the highest value, i.e. $V_R = c$. Remembering that A=0, Eq. (1) becomes

$$M = c^2 G^{-1} R/2. \tag{11}$$

From (10) it results

$$H = H_b(1+H_b t)^{-1} \tag{12}$$

where the integration constant H_b is the value of H at t=0, from which $H_0 \approx t_0^{-1}$ (H_0 and t_0 being the present value of H and the age of the universe respectively), as it should be expected. This result cannot be obtained from Friedmann's equation for any value of k if the gravitation term is different from zero. For k=0 and p=0, it would be $H_0 = 2t_0^{-1/3}$, a too low value.
Up to this point it has been irrefutably demonstrated that M varies linearly with time and no arbitrary hypothesis have been introduced for this demonstration, since it is based on generally accepted premises (a constant speed limit and the Hubble law). In the next section, in order to calculate the cosmological parameters, two additional assumptions will be introduced (the ultimate micro-particles and the proportionality between their frequency and their linear density leading to Eq. (16). These assumptions are strongly supported by the results obtained; anyhow, any objection to them would not affect what has been independently demonstrated in section **2:** the absence of gravitational effects in the expansion process and the variation of mass with time.

3. Derivation of the present and primordial cosmological parameters

It has been demonstrated above that the mass of the universe is increasing linearly with time and consequently the expansion speed is constant. From these premises it will be possible to derive theoretically the fundamental cosmological parameters with an accuracy of ± 0.12 per cent, using, as experimental datum, only the experimental relic temperature of the CMBR, 2.7255 ± 0.0006 °K (Fixsen, 2009), since the CMBR, predicted by Gamow (1948) and by Alpher & Herman (1948) and discovered by Penzias & Wilson (1965), is the most accurately known of the principal cosmological parameters.
Since the CMBR is in thermal equilibrium with the system and the temperature T represents the average kinetic energy in the system, it will be

$$kT = qc^2/2 \tag{13}$$

where k is the Boltzmann constant (1.3806503×10^{-23} JK^{-1}). The mass q may be regarded as the mass of a real or hypothetical particle, as though the ultimate structure of the substance constituting the universe consists of micro-entities of mass q. The final result will suggest whether we may accept these entities as really existing or just as a helpful formalism to allow the calculations.
From Eq. (13) and T = 2.7255 K, we obtain

$$q_0 = 8.374\times10^{-40} \pm 0,002\times10^{-40} \text{ kg.} \tag{14}$$

where q_0 is the present value of q.
Let us express the total energy qc^2 of each particle as $hc\lambda^{-1}$, from which

$$q \quad = hc^{-1} = 2 \quad m_p l_p \tag{15}$$

where h is the Planck constant and is a wave length associated with the micro-entity, as though these real or hypothetical micro-entities may have an undulatory behaviour. The frequency c is assumed to be proportional to the linear density of the Mq^{-1} micro-entities, $(3Mq^{-1}R^{-3}/4\ \ )^{1/3}$, from which, and from Eq. (11),

$(3Mq^{-1}R^{-3}/4\ \ )^{1/3}$ $= [3\ c^2G^{-1}R^{-2}q^{-1}/8\ \ \ ^{1/3}$ = constant
(16)

From Eq. (16) and (15) we derive that R and Rq^2 are constant. Therefore, it will be

$R = R_bq_b^2q^{-2}$ (17)

where R_b and q_b refer to the initial time zero.
Since q_0 is known from Eq. (14), it is possible to calculate the radius R_0 of the universe at the present time if we know the initial values R_b and q_b .
Putting $_b$ = 2 wR_b,, w being a proportionality factor (which we expect to be unity for analogy with m_pc^2=hc(2 $l_p)^{-1}$), Eq. (15) becomes

$wR_bq_b = m_pl_p$ (18)

Let us indicate as x^3 the volume containing one entity at the initial conditions and put

$x = f2\ \ l_p$ (19)

and

$R_b=gx = fg2\ \ l_p$ (20)

where f and g are proportionality factors which we expect to be very simple, as they are the dimensionless ratios of fundamental lengths.

From the density of micro-entities $3M_bq_b^{-1}R_b^{-3}/4$ and considering Eq. (11)

$(3M_bq_b^{-1}R_b^{-3}/4\ \ )x^3 = 3\ c^2G^{-1}R_b^{-2}q_b^{-1}x^3/8\ \ = 1$ (21).

Substituting in (21) R_b, q_b and x as derived from (18), (19) and (20), and $G=c^2l_pm_p^{-1}$ we obtain:

$f^2g^{-1} = 2(3\ \ w)^{-1}$ (22)

It will be shown that f = √2 and g = 3
Actually if we take $f = z\sqrt{2}w^{-1}$ and consequently $g = z^23\ \ w^{-1}$ we would obtain from (20) and (18)

$R_b = 1.3533\times10^{-33}\ z^3w^{-2}$ m $q_b = 2.5992\times10^{-10}\ z^{-3}w$ kg .

Then from Eq. (17) and (14):

$R_0 = 1.3040 \pm 0.0016\times10^{26}\ z^{-3}$ m and

$t_0=R_0c^{-1}= 4.3490\times10^{17}\ z^{-3}$ s = $13.793 \pm 0.016 \times 10^9z^{-3}$ yr

where t_0 is the age of the universe. Since 13.793×10^9 yr is very close to the expected value, not only z^{-3} must

be very close to unity but, if f and g, as said above, are expected to be very simple factors, it has to be z=1.
For the same reason, it should be decided on w=1. In any case, the value of w would affect only the primordial parameters, but not the present age or size The value of R_b would represent the radius of the primordial universe in case of spherical symmetry, or an average radius in case of a different form, e.g. a lenticular shape.
Since $R_b = 6\sqrt{2}\pi^2 l_p$ is about 10^2 larger than l_p, a quantum approach is probably unnecessary even at that early stage.
From Eq. (12), where $H_b = c\, R_b^{-1}$, H can be calculated as a function of time. At present it is $H_0 \approx t_0^{-1} = 2.299 \pm 0.003 \times 10^{-18}\, s^{-1} = 70.94 \pm 0{,}09\ Kms^{-1}Mpc^{-1}$. The values of R_0, t_0 and H_0 obtained in this study, depending on the value of l_p and the second power of q, are calculated with an accuracy of ± 0.12 per cent. The value of H_0 here obtained has therefore uncertainty limits definitely more narrow than those obtained from other sources. It is in agreement with the data of NASA's WMAP (Wilkinson Microwave Anisotropy Probe) (71.0 ± 2,5) and HST (Hubble Space Telescope) (70.6 ± 3.1), as reported by Larson et al. (2010) and by Suyu et al. (2010) respectively. It is also in agreement or compatible with the data from Friedman et al. (2001) (72 ± 8), Peacock (1999) (55-74) and Maoz et al.(1999) (81 ± 9), but not with the data from Willick & Batra (2001) (85 ± 5) and Riess etal (2011) (73.8 ± 2.4).
Being H=c/R, according to Eq.(11), we obtain at any time

$$\Omega = 8\pi G\rho H^{-2}/3 = 1 \tag{23}$$

From Eq. (11) the primordial equivalent mass M_b results to be $M_b = 9.113 \pm 0.007\ \mathbf{x}10^{-7}$ kg. The present mass is $M_0 = 8.781 \times 10^{52} \pm 0.020 \times 10^{52}$ kg and the total density is

$$\rho_0 = 3M_0R_0^{-3}/4\pi = 9.45 \pm 0.04\ \mathbf{x}10^{-27}\ \ kgm^{-3} \tag{24}$$

4. Further possible information from the model

The very good agreement of the results with the expected values, may suggest that the hypothesized micro-entities be a reality. If the q masses are a physical reality, the decrease of q with the square root of time may provide some information about the appearance of some elementary particles. Actually, it should be expected that an elementary particle of mass m_x can be present only if the mass q of the ultimate entities is lower than m_x. So electrons should make their appearance when (or after) q reaches the value of their mass i.e. at $t = 3.675 \times 10^{-1}$ s, muons at 8.596×10^{-6} s, tau particles at 3.039×10^{-8} s, Z and W bosons at 1.153×10^{-11} and 1.465×10^{-11} s respectively.
Since the baryonic density is estimated to be 4.5 per cent of the total matter, the appearance of DM cannot occur later than 6.2×10^{8} yr (4.5 per cent of the present age). At that time the value of q was 1.8×10^{-39} kg and this should represent the lower limit of the mass for the constituents of DM.

5. Conclusion

The proportionality between mass and size of the universe and the absence of gravitational deceleration have been demonstrated, based on the Hubble expansion and the existence of a constant limit speed, without the need of additional assumptions. The mass variation is at the origin of a negative pressure which makes the FLRW and the Newtonian equations converge to the same results.
It has been pointed out that this is not a steady-state model but a big bang model; therefore all the implications of a big bang model, the CMBR included, are compatible with it.
Considering the three cases a), b) and c) in section **1.** it was assumed that only one variable, M or V_R, may vary. An alternative would be that both mass and velocity increase with time (dM/dR>M/R), but, due to the requirement of V_R<c, this condition would alter only the initial stage of the expanding process. This alternative, which could be the object of future study for completeness, does not appear, however, very plausible in the light of the present results.
The values of the cosmological parameters at the present time have been derived using only one experimental datum, the CMBR temperature, and have therefore a much lower degree of uncertainty than those from other sources. The very good agreement with the expected values is a strong support for the model which has been developed. From this model the primordial values of the parameters have been theoretically calculated. The cosmological constant and the curvature of the universe are null.
The whole universe is now practically within the horizon, as only observers in a boundary shell, which cannot exceed $2R_b = 2.7 \times 10^{-33}$ m, would have not been in contact with the whole universe.

If the q entities were real, the cosmological process may be discontinuous if only integer values of Mq^{-1} (the number of particles) were allowed, but, since the points are very close to each other, the curve mass versus time can be considered as continuous and what has been described above maintains its validity. As the density and temperature are decreasing with the second power and with the square root of time respectively, some revision in the description of the past cosmological process may be necessary. This and other aspects will be the object of future work for a more detailed characterization of the model.

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